Instability and reconnection in the head-on collision of two vortex rings

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ONE mechanism by which fluid flows increase their complexity is through the instability of vortex filaments. When an instability brings vortex filaments of opposite circulation together, the filaments may break and rejoin in a process known as reconnection. This process of instability and reconnection leads to some fundamental changes in the topology of flows. Here we present experimental observations of a special type of instability in which two colliding vortex rings become unstable and reconnect to form a series of smaller rings. Although this phenomenon was briefly noted more than a decade ago[1], no detailed observations were made, and little is known about the mechanisms involved. We have used coloured dyes to reveal the detailed structure of the small rings and many other features, including a short-wavelength instability around the circumference of the colliding rings. At high Reynolds number, collision leads to a turbulent cloud, with the occasional appearance of small rings.

The experiment was conducted in water in a glass tank (1.22 m long, 0.36 m wide and 0.47 m deep) in which were immersed two horizontally opposed nozzles spaced 220 mm apart. The position of one nozzle could be finely adjusted to make the rings collide exactly head-on. Both nozzles were connected to a piston, which ejected short, equal pulses of water from both nozzles simultaneously to produce two identical vortex rings travelling towards each other. Accurate, repeatable results were achieved by driving the piston with an electronically controlled stepping motor: this meant that the circulation and Reynolds number of the rings could also be determined. The vortex rings were made visible by releasing neutrally buoyant dyes around the circumference of each nozzle; the resulting flow patterns were recorded using a video recorder.

Figure 1 shows different stages of a head-on collision for a Reynolds number (Re) of $\sim$1,000. (The initial Reynolds number of each ring is defined by UD/ν, where U is the initial translation velocity, D is the diameter of the ring and ν is the kinematic viscosity.) Figure 1*b* shows that when the two vortex rings are close to one another, the velocity induced by one ring on the other causes both rings to grow in diameter. The early stages of this growth follow the predictions of a inviscid analysis reasonably well, but when each ring has increased in size to about four times its initial diameter, a symmetrical instability in the form of azimuthal waviness begins to develop. As time progresses, the waves on the rings grow until they touch at the locations of maximum inward displacement. At the points of contact, the segments of the two vortex filaments eventually become interconnected to form small rings, a process commonly referred to as 'vortex reconnection'. As can be seen in Fig. 1*e*, the observation that each small ring is made up of both red and blue dye indicates that it consists of segments from both of the original rings (Fig. 2 shows a close-up view of the small rings). Throughout the process of reconnection, the original vortex rings continue to grow in diameter, albeit at a slower rate. This growth is associated with stretching of the contact regions between the waves, and seems to be related to the reconnection process. Once the small rings have fully formed, the original rings cease to exist, and the small rings then convect away radially from the central axis at slightly different speeds. The azimuthal waves that occur during the collision do not always form a regular pattern around the rings: the wavelength of the instability varies along the circumference and from run to run.

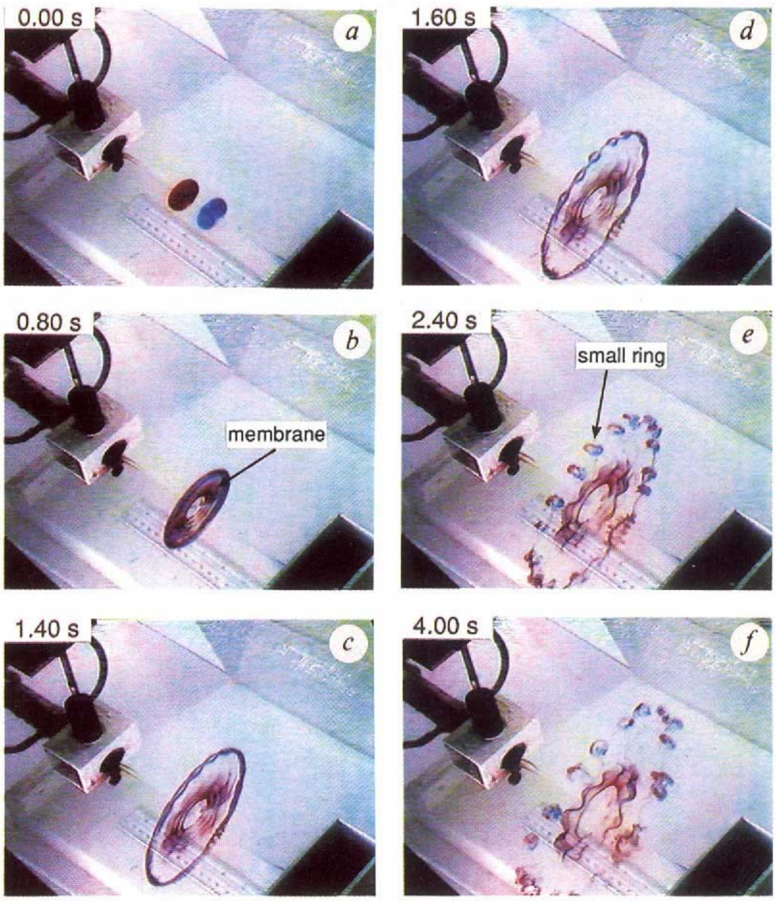

FIG. 1 A sequence of photographs showing different stages of the head-on collision between two identical vortex rings. The initial Reynolds number of each ring is roughly 1,000. The first photograph in the sequence, *a*, has been arbitrarily assigned as $t = 0.00$ s. The elapsed time for the subsequent stages of the collision is shown in each photograph, with different time intervals adopted to illustrate the main features of the flow. The 'membrane' structure with concentric ribs observed in *b*–*f* occurs because the vortex rings consist of rolled-up spiral dye sheets which become squashed and flattened during the collision. Each turn of the sheet forms a fold which becomes one of the concentric ribs on the membrane. Because of the effect of viscous diffusion, we do not believe that the membrane contains much vorticity.

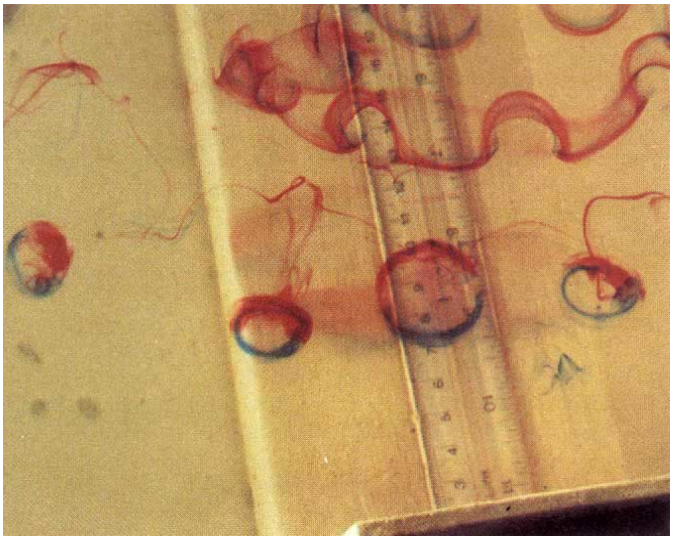

FIG. 2 Close-up view of the small rings.

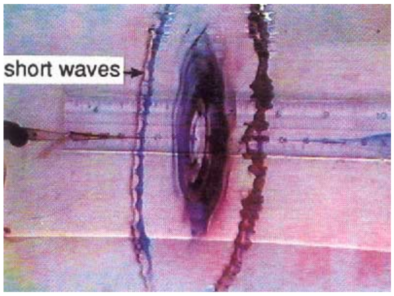

FIG. 3 Short-wavelength instability. This instability can also be seen on part of the rings' circumference in Fig. 1*c* and *d*.

In some cases the waves did not develop significantly over a considerable part of the circumference. Small rings have also been observed to form during the collision of offset vortex rings[2].

Unexpectedly, a short-wavelength instability also formed in some of the runs (see Fig. 3). To our knowledge, this is the first time that this type of instability has been identified in colliding vortex rings. On some occasions, both modes of instability coexisted on the same ring and did not interact noticeably (see Fig. 1*c*, *d*). The short-wavelength instability is similar in appearance to the 'core bulging' or 'bursting' instability observed in trailing vortices, which numerical analysis[3] suggests is caused by the effect of viscosity.

Figure 4 shows similar collisions at higher Reynolds numbers. For a Reynolds number between 1,000 and 1,500 (Fig. 4*a*), the general behaviour of the vortex rings is similar to that shown in Fig. 1, although the wavelength of the long-wave instability seems to have decreased. Furthermore, the short-wavelength instability discussed earlier did not occur in this case: this is consistent with the suggestion that the instability is viscous in nature, as at higher Reynolds number the effect of viscous forces

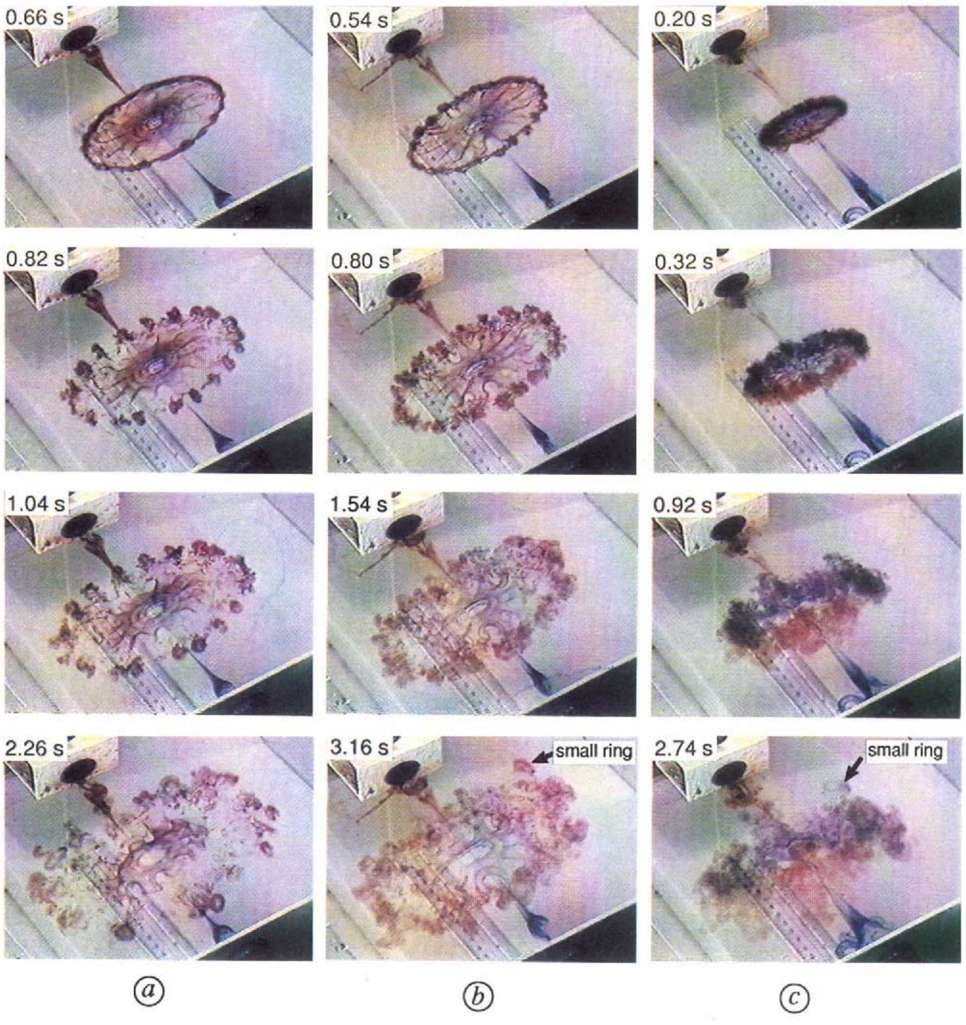

FIG. 4 Collision at higher Reynolds numbers. *a*, Re ≈ 1,450; *b*, Re ≈ 1,850; *c*, Re ≈ 3,500. The position of the rings at time $t = 0.00$ s is the same as in Fig. 1*a*.

becomes less important. As the Reynolds number is increased from ~1,500 to 3,000 (Fig. 4*b*), the flow goes through a transition in which the small rings appear to have fine structures superimposed on them. When the Reynolds number is increased beyond 3,000, the two rings disintegrate into a turbulent cloud as shown in Fig. 4*c*. This disintegration has also been observed by Schultz-Grunow[4]. In the two cases at higher Reynolds number, a single small ring may be seen escaping from the interaction region, even though the rest of the flows are complicated (see the final photographs in Fig. 4*b*, *c*). It may be that in some places around the circumference, the local conditions are more characteristic of a lower Reynolds number.

The development of the azimuthal waves and the eventual formation of the small rings is reminiscent of the interaction between the two trailing vortices produced by the wing tips of an aircraft (ref. 5). Using an inviscid model, Crow[6] analysed the growth of waves on the trailing vortices in order to predict the wavelength that would dominate. The similarity between the two flow cases and the noted inviscid nature of the early stages of the interaction prompted us to attempt to apply this model. In this approach, the size and separation of the cores determine the wavelength of the instability, which increases with increasing core size and decreasing separation (although the dependence on the core size is weak). In this case, the core size is estimated using the circulation and measured translational velocity, and the core separation determined from a video of laser cross-sections of the flow (not shown here). In Crow's model there is no stretching of the filaments and the wavelength of the instability does not change with time. In our case, the wavelength of the instability increases as the rings stretch, although the number of waves on the circumference remains constant. Thus as a first approximation, we ignore the stretching and consider that the instability forms at the instant when the azimuthal waves first become apparent on the circumference of the rings. From the values of the core diameter and separation at that instant, the wavelength of the instability can be determined using Crow's model, and the number of small rings that should form can be estimated by dividing the circumference of the vortex rings by the calculated wavelength. Given that the ratio of the radius of curvature to the core separation is ~50, the curvature of the filaments can be neglected and they are treated as a simple trailing vortex pair. Within the accuracy of the measurements, the predicted number of rings (N) falls between 11 and 24 for all Reynolds numbers in the range 860–1,500, in rough agreement with the experimental observations (on average, N increases from 15 to 20 as Re increase from 860 to 1,500). Although it is difficult to measure the circulation and core separation very accurately, the agreement of this first approximation with observation suggests that further investigation may be worthwhile.

It is important to note that the analysis of Crow[6] breaks down when the filaments touch, and can therefore only explain the initial growth of the waves and not the formation of the rings. The rings are formed by the breaking and rejoining of the vortex filaments, a process commonly referred to as 'reconnection'. This has been the subject of much recent study, and although not all researchers agree as to the detailed mechanism involved, the basic overall processes they discuss are similar. Briefly, when two vortex filaments are brought into contact by an induced velocity field, viscous diffusion in the contact region causes annihilation of vorticity. The annihilation of vorticity effectively 'severs' the filaments, and, due to the kinematic constraint that vortex lines cannot end inside a fluid, they reconnect on either side of the contact region. For a summary of different views of the detailed mechanisms involved, see refs 7–10. □

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Coherence of the superconducting wavefunction between the heavy-fermion superconductor UPd_2Al_3 and niobium

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HEAVY-fermion superconductors (in which the charge carriers have an effective mass ~100 times the free electron mass) have been the subject of intense study during the past decade[1], in part because of the suggestion that some of these materials may exhibit non-conventional pairing mechanisms[2]. Here we report a demonstration of quantum coherence of the superconducting wavefunction between the conventional superconductor niobium and the recently discovered[3] heavy-fermion superconductor UPd_2Al_3, which is a potential candidate for a non-conventional pairing mechanism[4]. The experimental method is similar to our earlier work on high-T_c materials[5,6]: we use a small pointed rod of UPd_2Al_3 to bridge the gap in an almost closed niobium ring. We observe persistent currents in the composite ring, and trapped flux, which is in discrete quantum states separated by the flux quantum $h/2e$. Although the observation of phase coherence between UPd_2Al_3 and niobium may not constrain the nature of the pairing in UPd_2Al_3, we also observe Josephson-like current–voltage characteristics at the junction, but with very small products of critical current and normal-state resistance. If other possible causes can be eliminated, these small I_cR_n values may point to a small tunnelling probability between the two superconductors, and hence to unconventional pairing in UPd_2Al_3.

In a conventional BCS superconductor the electrons are paired in a spin singlet *s*-state. It is in principle possible to have other forms of pairing (*p* or *d*), and there has been much discussion as to the possible difficulty in obtaining coherent tunnelling of Cooper pairs between systems with superconducting states of different symmetry (see for example refs 6–8). It is now generally believed that the symmetry-breaking effect of the barrier allows coupling between states of different orbital angular momentum, and that coherent tunnelling between states with singlet and triplet pairing is also possible, albeit with reduced matrix elements[7]. The possibility of superconducting states involving quasiparticles with even more exotic symmetry, such as anyons, has recently excited interest[9], but the consequences for coupling across a barrier between anyonic and non-anyonic superconductors are unclear. In any case, demonstration of coherence in systems having the potential for mixed symmetry is of fundamental importance.

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